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Metaphors of hierarchy in mathematics education discourse: the narrow path

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This paper adopts a rhetorical perspective in order to examine language about children in the discourse of mathematics education through a study of metaphor. Previous research has tended to emphasize the notion of 'beliefs', which locates responsibility for problematic conceptions of children within the heads of individuals, particularly practising and preservice teachers. Using the notion of metaphor, this paper examines several texts in US mathematics education, including conversations in an elementary classroom, a university mathematics methods classroom, mathematics textbooks, and standards documents. All of these texts draw on the metaphor of children's learning as travel along a physical path, which supports talking and thinking about children in hierarchical ways. The dominance of this metaphor presents a new challenge for teacher educators concerned with equity: that of examining their own language and practices for hierarchical language.

Keywords: equity; discourse; mathematics education; metaphor; teacher education.

One afternoon during the 2005–2006 academic year, as part of a research project on learning to teach mathematics, I accompanied a preservice teacher I had been observing on a field trip to Northside Elementary School[1] for her university methods class. Usually I saw Sara in her grade 3 placement classroom in Blythe Elementary School, an urban school, that served primarily low-income and minority students. Sara, with the enthusiastic support of her mentor teacher, Diana, had been working hard to incorporate reform-oriented strategies, such as discussions and problem-solving, into her teaching.

The field trip to Northside Elementary School had been organized by the university faculty member teaching Sara's methods course, which she attended during the one day each week she was not in the elementary classroom. David, the mathematics methods instructor, wanted to model reform-oriented teaching in one of the elementary schools in which his university students were placed. He arranged to teach lessons in a grade 4 and a grade 1 classroom in a suburban school located near the university. Sara and I visited Northside Elementary School only on the day of this university field trip.

The grade 4 room, which was large enough to accommodate easily the 21 visiting pre-service teachers as well as the 22 children, had been arranged

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with the desks in groups of four-to-five, angled toward the white board at the front of the room. David began the lesson by telling the children that definitions were important in mathematics and asking them to help him come up with a definition for a chair. Students called out characteristics, and David challenged them through the use of examples and counter-examples until the class agreed on a list of minimally defining characteristics (i.e. has legs, has a back, you can sit on it, is manufactured). David then led them through similar exercises with squares, rectangles, parallelograms, and rhombuses, and finally asked them to state relationships between these figures (i.e. a rectangle is a special kind of parallelogram). After the lesson, the interns gathered in the hallway to wait until the end of recess, when they would go to see David's lesson in the grade 1 room.

While waiting, Sara approached Jack, an intern assigned to a another grade 3 classroom at her elementary school, and remarked, 'Can you picture one of our kids saying the square is a polygon? They don't even know what that means.' Jack said 'No', and Sara turned to Katie, another intern from her school, and continued:

> This just shows me how far behind our kids are. I mean, they're 3rd grade, but they won't be here next year. Diana is always saying our kids are so good, but they're not like this. Even when things are going well, when they're excited about something, they just don't have the attention span.

Katie agreed, saying: 'They don't raise their hands. Or rather they blurt out first and then they raise their hands.' Sara nodded.

This episode struck me because I was surprised by Sara's comments—first, because she had always seemed to show a great deal of pleasure and pride in the accomplishments of the students in her classroom, and, second, because her and Katie's perceptions of the lesson that David had taught were so different from mine.[2] I had observed several students who spoke out without being called on; in fact, David had seemed to encourage this practice by asking questions of the whole class and waiting for a choral answer. In addition, although the class had produced a great deal of technical vocabulary, including 'polygon', 'quadrilateral', and 'symmetrical', most of these words had come from just a few students. In a lesson I observed Sara teach in her classroom the following week, her students used the words 'square number', 'repeated addition', and 'strategy' while describing their work. These words may not have been as unusual as the geometric ones, but they showed appropriate uses of mathematical vocabulary for a grade 3 unit on multiplication. Yet, Sara was clearly struck by differences between her class and the suburban class she observed David teach. And she did not see these differences as the results of participating in a different kind of lesson with an experienced mathematics educator, of being a year older, or of studying a different content strand within mathematics. Instead, she saw the differences as innate in the students, calling her own class 'behind', 'not like this', and lacking in 'attention span'.

My meditations on Sara's comments led me away both from my original research questions and my social-science methodologies. For the most part, studies of such comments as Sara's that draw on social-science traditions have focused on the individual beliefs of participants. For reasons I will explain in the following section, I did not want to code and categorize the comments of Sara and her classmates in order to document their problematic beliefs. Instead, I wanted to trace comments like Sara's through a variety of texts—both written and spoken. In other words, I wanted to make a linguistic turn. Fendler (2006) has suggested the field of rhetoric as one possible alternative to social science in the analysis of teaching and teachers. Following this tradition, analysis can examine persuasive techniques, such as metaphor, simile, and analogy, as a way of calling attention to dominant ideas in a discourse, rather than looking closely at human interactions in social settings. This move toward rhetoric, particularly to belief and metaphor, allowed me to take the focus off Sara as an individual and to examine the mathematics education community more broadly.

Problematic beliefs

The US literature in mathematics education, and in teacher education across disciplines, has often explained episodes such as the one I described above as instances of beginning (or experienced) teachers revealing their problematic beliefs. For instance, in this literature teachers have been described as holding beliefs that embody 'dysconscious' racism (King 1991), that promote colour-blindness (Bell 2002), and that link low achievement to inadequacies in the students' culture (Cooper and Jordan 2003). In a review of research on prospective teachers' beliefs about teaching children of different races, ethnicities, and socio-economic backgrounds, Gomez (1993) described a host of problematic teacher beliefs that have been documented in dozens of studies, including beliefs that economic rewards are fairly distributed, that low-income families do not support their children's learning, and that some students cannot learn.

Most studies of teachers' beliefs in mathematics education have focused on teachers' beliefs about how mathematics should be taught or about what mathematics is, rather than on beliefs teachers hold about students.[3] However, a few studies (Fennema *et al.* 1990, Tiedemann 2002, Sztajn 2003) have looked at interactions between these belief systems. For example, Sztajn (2003: 53–54; emphasis added) noted that teachers 'have to put together, according to *their own* beliefs and interpretations of existing rhetoric, what they consider to be the best mathematics education for their students.' To do this, she contended, teachers draw on both their sense of what mathematics is and on their beliefs about who the children in their classrooms are. Based on a case-study analysis of two experienced teachers, she concluded that the teachers' beliefs about mathematics and children caused them to use drill-oriented work with classrooms of low-income students and to assign activities that promoted higher-order thinking to classrooms with children from more affluent families.

Framing teaching decisions and comments like Sara's as instances of problematic beliefs has meant that the interventions proposed by teacher educators have often focused on finding ways to successfully change individuals' beliefs.[4] For example, in their study of mentor–novice pairs, Achinstein and Barrett (2004) discussed beginning teachers' tendencies to see their diverse classrooms as collections of management problems. The authors of the study described ways that mentors were able to help beginners reframe their thinking about students to include political and social considerations, which caused the novice teachers to change their beliefs about their students' academic capabilities. Other studies have focused on identifying qualities in beginning teachers that lead to either 'problematic' or 'productive' beliefs, such as previous experiences with cultural diversity, empathy for others, or valuing of diversity (Pohan 1996, Smith *et al.* 1997, Garmon 2004).

My concern with framing comments like Sara's—as well as questionable teaching practices—as the products of problematic beliefs is that this theoretical stance places too much emphasis on the individual. Although most studies acknowledge that individuals' beliefs are situated in larger social contexts, such methods as surveys and interviews work to highlight the responsibility of single individuals. Similarly, interventions that seek to change the beliefs of individuals through classes on cultural diversity, field experiences, or reflections on experience make what goes on inside individual heads the primary locus of change.

I see two difficulties with this model. In practical terms, it is slow work. Following this model, teacher educators must diagnose and treat each prospective teacher they encounter from now until the end of time. Theoretically, this model is also troublesome. Whether one follows Vygotsky (1978: 88; emphasis removed), who wrote that learning 'presupposes a specific social nature', Bakhtin (1981: 294), who said that the word 'exists in other people's mouths, in other people's contexts, serving other people's intentions', or the host of other thinkers who have contended that researchers cannot consider the individual without the social,[5] studies of beliefs seem to give far too much weight to the individual, both in terms of responsibility for problematic ideas and in terms of proposed interventions. Blake *et al.* (1998: 88), in one of the most eloquent articulations of this idea, wrote that:

> In part what is given in teaching, in the initiation into a culture, is a gift that cannot be refused. What we come to know in this way precedes the possibility of our autonomy.

Sara's comments comparing the children at her urban school with those she saw in the suburban classroom did not reveal her *own* beliefs; rather, the way she came to see her children as 'behind' preceded the possibility of her autonomy.

Dense metaphors

Rather than analysing Sara's comments as indicators of her beliefs, I want to examine Sara's words as re-articulations of a common metaphor in the current discourse about children. My hope is that by approaching the analysis in terms of metaphor, I will be able to consider simultaneously the actions of individuals and the 'social soup'[6] in which those actions are located. To do this, I draw on the notion of metaphor articulated by Lakoff and Johnson (1980), who showed that metaphors are more than ways of using language, but are also ways of understanding concepts. They used the metaphor of 'argument as war' to demonstrate both the pervasiveness of metaphorical language in everyday communications in the US and also the ways in which metaphors can influence how people see the world. Lakoff and Johnson pointed out that in US culture people talk about 'attacking', 'demolishing', 'shooting down', and 'winning' arguments. They plan 'counter-attacks' and 'defend' their points. In contrast, the authors asked readers to imagine a culture where 'argument as dance' was the dominant metaphor for speaking and thinking about disagreements:

> [T]he participants are seen as performers, and the goal is to perform in a balanced and aesthetically pleasing way. In such a culture, people would view arguments differently, experience them differently, carry them out differently, and talk about them differently. But *we* would probably not view them as arguing at all: they would simply be doing something different. (p. 5)

Using this example, Lakoff and Johnson suggested that metaphors are not simply instances of poetic language that writers use to make their work more aesthetically pleasing, but are ways of seeing the world that become more and more dense as they are re-articulated by multiple speakers. These dense metaphors then become part of everyday language and are often not registered as metaphors by listeners. For example, when Sara—and others—remark that some children are *behind*, we as listeners hear this metaphor literally. Rather than consider the word 'behind' to be a poetic description of children's academic performances, we interpret it as a real description of children's mathematical competence—no more metaphorical than remarking that some students scored 10 points lower on a test.

Building on Lakoff and Johnson's work, Santa Ana (1999), who examined the prevalence of the metaphor 'immigrants as animals' in the media, maintained that the more commonplace metaphors become and the more prosaic they seem, the more powerful they become in shaping the way people think. He wrote:

> When an original, truly novel metaphor is used, the reader of the turn of phrase is prompted by its novelty to evaluate the metaphor for its appropriateness, creativity and utility. The mindful reader can choose to reject the linkage. If, however, the metaphor does not draw attention to itself, then the reader is most often unaware that a conceptual linkage has been reproduced and is being reinforced. (p. 217)

Lakoff and Johnson (1980) suggested that many of our most common metaphors—those that do not 'draw attention' to themselves—are based on our physical experience of the world and can influence the meaning of other metaphors that draw on the same physical experiences. Many concepts we talk about metaphorically are structured in terms of up and down, and forward and behind. That is, these metaphors draw on our notion of a physical path that can be travelled in two directions, either vertically or horizontally, in order to explain some other concept. For instance, Lakoff and Johnson wrote that 'conscious' is often portrayed as 'up' (wake up, early-riser, up and at 'em) whereas 'unconscious' is often portrayed as 'down' (fell asleep, dropped off, sank into a coma).

Coherence among metaphors can be created when multiple metaphors draw on similar physical concepts. For example, many concepts (e.g. 'good', 'more', 'awake', 'happy') map onto 'up', whereas other concepts (e.g. 'bad', 'less', 'asleep', and 'sad') map onto 'down' in many commonly used expressions (Lakoff and Johnson 1980). Sfard and Lavie (2005) offered one empirical example of this kind of coherence when they told the story of two 4-year-olds who insisted that the mystery box they chose was 'more' than an identical rejected box. When questioned, the children agreed that the two boxes were the same size; however, they stood by their claim that the chosen box was 'more huge'. The adults present could not understand the children's statements, but Sfard and Lavie concluded that the children associated the word 'more' with 'good' and used it because they believed that the chosen item must be by definition 'more' than the less desirable item. This is an example of coherence between the concepts of 'good' and 'more', both of which are often discussed in ways that draw on the physical concepts of 'up' and 'ahead'.

Talking about children: metaphors of hierarchy

In order for Sara to offer her relatively straightforward comment that watching the children in the model lesson made her realize 'how far behind our children are', she needed to make several decisions: first, that an appropriate action when observing two sets of children is to compare them; second, that an appropriate next step is to rank them according to some criteria; and third, that an effective way to express that ranking is to use a metaphor that draws on front-to-back directionality. Neither Jack nor Katie, the interns to whom Sara spoke, seemed perplexed by her use of the word 'behind'. They did not seem to stop and evaluate the metaphor, as Santa Ana (1999) suggested that people may do when confronted with novel metaphorical language. Both seemed to understand that Sara was ranking the children in some way, and Katie, at least, seemed to agree with the ranking, offering her own evidence to support it: the children in her urban classroom called out without raising their hands. In this section, I would like to examine the prevalence of metaphors of hierarchy in discussions about children in mathematics education literature by examining a variety of sites, including elementary mathematics textbooks, US standards documents, conversations in David's methods classroom, and conversations in Sara's elementary classroom.

In the methods classroom, both David and the other interns often talked about students in ways that created hierarchies by drawing on metaphors of the physical world. For example, at various times in the semester, the following comments were made:

> What do you do with a 2nd-grader who's really awesome in math—who's really *far ahead* of everyone else?
>
> She's in special ed. She's really *low*.
>
> Obviously you need to set up groups of kids in problem-solving that can help each other and *pull people along* who are having trouble.
>
> Some of your kids—they're going to be *way up here* (holding hand above head). You have to *rise up* to meet them.

> I'm afraid he'll have to be retained. He can't *keep up*.

Each of these comments draw on a notion of children ordered in physical space, whether horizontally ('far ahead') or vertically ('low'). Some of the comments put students in motion. The line of students is marching forward and children must either 'keep up' or be 'pulled along'. This is drawing on a similar sense of space as Sara's comment that her children 'won't be here next year'. In both cases, the mathematics is portrayed as a path that must be travelled. Sara is saying that her grade 3 students will not have made enough progress along the path in order to be in the same place as the grade 4 students in a year's time. The remark about 'pulling children along', which was made by David (the methods instructor), implies that some children will not be able to travel the path under their own power and must be dragged forward by their more able classmates. Initially, it may seem strange to think of these ordinary bits of language as metaphors; however, like more poetic figures of speech, each of these phrases asks the listener to think of children in non-literal ways. Children are not really ordered in physical space. There is no path. No one (contrary to contemporary US political rhetoric) is actually being 'left behind'. As Santa Ana (1999) has pointed out, one of the reasons this sort of language seems unremarkable is because it is so common. Sara encountered these kinds of physical metaphors for children in her elementary classroom, in her textbook, and in documents from the state.

For example, in one lesson Diana (the classroom teacher) said: 'Ben, you did it in a really sophisticated way, but maybe you can *bring your thinking back* to other people'; 'Come on, Jerome! You need to *keep up* with the rest of the class'; and (to the whole class) 'You're letting Tyler and Ben *carry you*'. This last variation echoes David's comment about 'pulling kids along'. The children in Diana and Sara's class also used these kinds of metaphors. During independent work, Aliah remarked that 'Mia is *ahead of me*. She knows her division and her multiplication.' Once, Sara even physically acted out the metaphor of mathematical achievement as physical ordering on a path by calling children to line up to go to lunch in an order that ranked their participation in the lesson. Students who spoke frequently were at the front of the line, while Mia, who at that point in the year spoke very little English, brought up the rear.

Although the language in state documents and mathematics textbooks tended to be more formal than the spoken language I have sketched, examples of phrases that drew on metaphors of crossing physical space to talk about children's learning of mathematics did occur. In a handout for parents about the state mathematics standards in grade 3, the authors wrote,

> The expectations were designed to ensure that students receive *seamless* instruction from one grade level to the next, leaving *no gaps* in any child's education. (Michigan Department of Education 2005b)

As with the phrase 'achievement gap', the language in the parent handout implicitly draws on the crossing of physical space as a metaphor for children's learning in mathematics, where gaps in the path can be seen as dangerous. The metaphor of physical space was also implicitly drawn on in the methods textbook that Sara often used when planning lessons. Van de Walle (2007: 95; emphasis added) wrote 'It remains true that students *will* *rise or fall* to the level of our expectations'. Here students are compared to an independent criterion (expectations) rather than each other, but still moving up is good and going back, or falling, is bad.

All of the metaphors discussed above hang together coherently because each uses the idea of a line in physical space, where being closer to the front or the top (or moving in that direction) is good. Lakoff and Johnson (1980) have argued that because human beings live lives as physical bodies moving through space, they have a tendency to frame abstract concepts—such as learning or mathematical ability—in terms of the physical world. Thus, the physics of moving along a path structures in particular ways our thinking about children and their learning of mathematics. Although saying a student is 'really far ahead' of everyone else hardly seems poetic, like other metaphors, it connects one concept (knowing mathematics) to another dissimilar concept (being physically in front). And, like all metaphors, this connection makes some features of knowing mathematics more salient than others. Repeatedly using language that describes children's learning of mathematics as a journey or a ranking in physical space emphasizes the idea that students' thinking, learning, and progress can be compared to each other. Just as location or progress along a path can be identified through the use of landmarks, children's mathematical progress can be identified by where they stand relative to each other or by where they stand relative to the content. This is how children can be thought of as 'behind' or 'ahead'.

These metaphors also, in effect, portray mathematics as a narrow path, which can be travelled in only one direction (up and ahead). Sara observed David teach a geometry lesson in a grade 4 classroom and said that her children were 'behind' the ones she taught each day. Geometry had not yet been taught in her classroom; yet Sara assumed that the children's performance in this geometry lesson revealed their understanding of other mathematical strands as well. She did not say, 'Boy, they know a lot of geometric vocabulary'. The ease with which she could perform the ordering of the children in these two classes and the ease with which her ordering was accepted by the other interns in part relies on the metaphor of mathematical achievement as a journey along a narrow path. One cannot simultaneously be 'ahead' in geometry and 'behind' in number. Each person has one location on the path.

The metaphor of travel along a path makes it difficult to see a student who struggles with number as mathematically competent because of an ability to visualize three-dimensional objects. This metaphor obscures multiple-entry points into mathematics and makes it difficult for teachers to see student work as varied without also seeing it as ordered. On a path, one is always nearer or farther from the front. Similarly, this metaphor can cause teachers to interpret chunks of mathematical content as necessarily ordered. In the methods class, one intern said that she could not move on to multiplication because many of her students could not subtract with regrouping. David asked what one had to do with the other and the intern replied, 'Subtraction with regrouping comes first'. Subtraction with regrouping did come first in the mathematics books used in Sara's classroom (as in most US grade 3 mathematics books); however, there is no mathematical reason to assume that competence in the former is a prerequisite for competence in the latter. Children's performance in different mathematical strands—such as geometry, measurement, data, and operations—can be quite varied, but using a metaphor that portrays mathematics as a 'seamless' path with no 'gaps' can make it difficult to recognize this fact.

The metaphor of the path also contributes to understanding children in ways that promote the idea that some children are good at mathematics and some children are not. When mathematics is seen as a linear list of skills that must be mastered one after the other so as to progress toward a single goal (the end of the path), then some children must always be closer to that goal and others must be further away. When Ben offered a new solution to an addition problem, Diana did not merely identify the solution as different, but labelled it as 'sophisticated', and told Ben to 'bring your thinking back' to the rest of the class. Similarly, students who are in different places than their classmates in terms of understanding mathematics are seen as problems for the teacher and other students. They must be 'pulled along' or work to 'keep up'. The metaphor of travelling along a path obscures ways of understanding student differences as interesting, valuable, or natural.

These ways of thinking are reinforced in mathematics education by many concepts that may not use words which explicitly refer to positioning in the physical world, but which nonetheless draw on the root idea that learning mathematics has one path which individuals are more or less well suited to travelling. In mathematics education, the notion of 'development', which underlies theories and practices that expect all children to progress through identical stages in identical orders (like travel along a path), is present in much of the writing about children. Piaget's theory that children develop the abilities to realize that objects do not disappear, to use symbols, and to conserve mass (among other abilities) at particular ages and in a predicable order (Piaget 1952, Piaget *et al.* 1999) has been particularly influential. David mentioned Piagetian concepts several times during his lectures, suggesting that the prospective teachers use simple activities, such as asking children to identify the larger of two sets of identical but differently spaced counters, in order to make decisions about what sort of instruction might be appropriate.

Similarly, the van Hiele levels of geometric thought, about which David lectured in class, and to which the author of Sara's methods textbook devoted nearly 40 pages (van de Walle 1997: 345–382), are grounded in a notion of development in which all children are seen as progressing through the same levels in the same order. Calling the van Hiele theory 'the most influential factor in the American geometry curriculum', Van de Walle (2007: 409) described the theory as saying that 'the levels are sequential. To arrive at any level above level 0, students must move through all prior levels'. In elementary school, children are expected to move from Level 0, where they can identify shapes only because they look just like other shapes they have seen ('It's a square because it looks like one'), to Level 2, where they can reason about the properties of shapes ('Because it's a square, all the angles must be 90 degrees'). Both the van Hiele levels and Piagetian stages support ways of thinking that allow some children to be seen as 'behind' and others to be seen as 'ahead', relative either to their classmates or to benchmarks delineated in the theories. In fact, this happened in the methods class on the day of David's van Hiele lecture when one of the interns volunteered that her daughter's kindergarten homework had been to name the features that made a circle a circle. David replied, 'I don't think kindergartners or even most first-graders could do that'. The intern responded that her daughter had said that circles do not have corners, that they go all the way around, and that they don't have straight lines. David shrugged, adding 'She must be very advanced'. Here, a 5-year-old's ideas about circles were labelled as advanced because the uptake of the van Hiele levels has been that kindergartners are focused on visualization. The theory created the opportunity for this child to be seen as 'ahead' and at the same time created opportunities for other children to be seen as 'behind'. My goal here is not to argue that the van Hiele levels are inaccurate, or that David's use of them was incorrect, rather that the prevalence of developmental thinking in the mathematics education discourse reinscribes the metaphors of hierarchy on which Sara drew to talk about her children.

In similar ways, the current system of organizing students in classrooms by age reinforces thinking about mathematics as a narrow path. Michigan's standards in mathematics:

> provide a set of clear and rigorous expectations for all students and provide teachers with clearly defined statements of what students should know and be able to do as they progress through school. (Michigan Department of Education 2005a)

In other words, these standards define the path; they lay out what a grade 3 student must do and make it possible for teachers to identify children for remediation (those who are behind) and enrichment (those who are ahead). As Fendler (1999) has pointed out, this is a different enterprise than looking at actual grade 3 students to figure out what it is they know and are doing. In the first case, children are judged relative to the developmental expectations. In the second, children can be seen as empirical evidence that the developmental expectations are misguided.

The metaphor of schooling as a path with grade-levels as landmarks dominates not only practices in the classroom—Diana and Sara used textbooks written for grade 3 students, prepared for the grade 3 standardized test, and followed the grade 3 standards—but also frames how educators talk about children. Diana frequently asked her students to 'act like', 'write like', and 'sit like' grade 3 students. On one particularly noisy afternoon, she looked slowly around the classroom until students quieted. Then she remarked. 'Now it's quieting down. That's good because I was a little worried. I was thinking somebody else might have snuck in here.' Concerned, Marcus asked, 'Who'? Diana said, 'Someone who wasn't quite ready to be in 3rd grade'. Jerome, who was frequently chided, asked, 'Like me?' Jerome had gone through more than 3 years of schooling in the same building with many of the same classmates. He was 8-years old and knew himself to be in grade 3. Yet, the frequency with which his behaviour was corrected and his answers revealed to be incorrect caused him to identify himself as someone the teacher thought was not ready for grade 3. In this instance, Diana reassured him that he was indeed ready, but Jerome seemed not only to recognize the metaphor of schooling as a path, but also to be able to identify his place along that path.

The grade 3 mathematics textbook (Andrews *et al.* 2004), which Sara used occasionally, also drew on this metaphor. Each page in the mathematics book represented one lesson. Students were expected to proceed along a path from the first page to the last. In addition, each page in the book came with three additional worksheets, labelled 're-teaching', 'practice', and 'enrichment'. Teachers were expected to assign each student one of these pages (based on the student's position relative to the intended lesson), and were told never to assign students more than one of these pages (students could not be in two 'places' at once). Each page of the mathematics book also had a section called 'quick review', which students in the classroom completed at varying rates. This was a constant source of frustration to Sara, who once exclaimed: 'It's Quick Review! What's it supposed to be? Quick!' Students, who were all supposed to be at the same place along the path, were not expected to take different amounts of time to complete the same task.

Coherence among metaphors; or what else can we think of as ordered?

In the previous section, I argued that the metaphor of travel along a narrow path saturates the discourse of mathematics education in language, materials, and practices. The prevalence of this metaphor creates certain discursive possibilities and closes down others. In other words, Sara's choice to compare and order the children in her placement classroom and the children in the model-lesson classroom emerged from a discourse community in which hierarchical thinking about children was common. Sara's adoption of that language can be seen as evidence of her participation in a community that structures thinking about children's learning in terms of the physical world.

However, the prevalence of hierarchical thinking does not explain the order in which Sara ranked the classrooms. Why did she see the students in her own classroom as behind, rather than ahead? In this section I argue that Lakoff and Johnson's concept of metaphorical coherence played a significant role in the order of Sara's rankings. When many different abstract concepts (such as 'good', 'happy', and 'conscious') are mapped onto the same physical location ('up') through the reiteration of metaphors, these concepts come to be seen as related, even though no actual relationship exists. In similar ways, the discourse in mathematics education links the abstract concept of *behind* with other abstract traits frequently used to describe children. The result is that some children are more likely than others to be seen as inadequate.

Sara's class at Blythe Elementary School, a small urban school, consisted of 19 students. Four of these students had been identified as White/Caucasian on school enrolment forms. The other 15 students had been identified as African American, Asian, or bi-racial. Many of these children had brown skin and brown eyes. In addition, many of the children of all ethnicities spoke non-dominant forms of English. In some cases, this was the result of English being a second language; in other cases, the result of speaking a dialect. For some students, the variations in their speech were quite subtle—such as slight southern accents or using words like 'y'all'. For others, variations included grammar often deemed unacceptable in school, such as ‘He don’t know’.

In the class of 22 students at Northside Elementary School which Sara visited on a one-day field trip with her university mathematics methods class, most students (about three-quarters of the class) had the pale skin and facial features that most residents of the US would probably identify as White. All the children who spoke used variations of English that would probably be considered standard. That is, they used contractions frequently, but used grammatical constructions that would be considered correct in formal, written English. Most children’s accents and inflections mirrored those of David (and Sara and me). In addition, children in the Northside classroom were more likely than children in Sara’s classroom at Blythe to wear clothing with labels recognizable as both fashionable and expensive.

In the preceding paragraph, I sketched out a few of the differences between the classes at Blythe and at Northside. Of course, there were many other differences. The architecture of the buildings and arrangement of furniture in the rooms were different. As a group, the children at Northside seemed slightly taller. The children at Blythe tended to talk to each other, whereas the children at Northside tended to address only the teacher. However, I did not randomly choose the differences I highlighted in the previous paragraph. I described them because I believe they were differences that Sara noticed and drew on in making her assessment. I would now like to examine some examples of discourse about children from the mathematics education community with the goal of demonstrating how the ethnic, racial, and class differences I identified above may have become salient for Sara and may have contributed to her ranking the two classes in the order she did because of a metaphorical coherence that has been created between descriptions of race and class and descriptions of mathematical ability.

More than a decade ago, the National Council of Teachers of Mathematics (NCTM) (1991: 4) issued what it called its ‘Every Child’ Statement in order to underscore the Council’s commitment to the ability of all children to learn mathematics. The second half of the statement may be summarized as follows (Van de Walle 2007: 95):

> We emphasize that ‘every child’ includes:
>
> - learners of English as a second language and speakers of English as a first language;
> - members of under-represented ethnic groups and members of well-represented groups;
> - students who are physically challenged and those who are not;
> - females and males;
> - students who live in poverty and those who do not; and
> - students who have not been successful and those who have been successful in school and in mathematics.

The order of the subjects in the above bulleted list is not coincidental. In the first position are the kinds of students who have trouble learning mathematics: speakers of English as a second language, minorities, those with disabilities, girls, the poor, and the unsuccessful. These statements would not have what Lakoff and Johnson (1980) called ‘coherence’ if they were all jumbled about.

The category of 'males' does not go with the category of 'poor' and 'minority'. Although the language of ordering was not used explicitly, the pattern of the bullets implies a continuum of students who range from 'those who have not been successful' to 'those who have'. In addition, ordering the subjects of each bullet in this way works to attach the categories of minority, female, poor, and physically challenged to unsuccessful.

Adding It Up (National Research Council 2005), a report on elementary mathematics in the US that was written by prominent mathematics educators, also invoked metaphors of hierarchy and linked notions of race and class with the ordering. First of all, the section of the report that addresses issues of equity is called 'Equity and remediation'. Immediately, this phrasing invokes the metaphor of the narrow path. Remediation is only possible when some students are behind. The challenge of equity becomes catching these students up to where they are 'supposed' to be, rather than on, say, learning to value where these students are. This section of the report goes on to say:

> A number of children, however, particularly those from low socioeconomic groups enter school with specific *gaps* in their mathematical proficiency. … Overall, the research shows that *poor and minority* children entering school do possess some informal mathematical abilities, but that many of these abilities have developed at a *slower rate* than in *middle-class* children. The immaturity of their mathematical development may account for the problems *poor and minority* children have understanding the basis for simple arithmetic and solving simple word problems. (pp. 172–173; emphasis added)

The authors of this paragraph explicitly invoked metaphors of travel along a path with the use of the words 'gaps' and 'slower rates'. In addition, they drew on notions of development by referring to the 'immaturity' of some students. Not only does the phrasing of this paragraph support ways of thinking that place some children ahead of other children, but the words chosen also attach demographic characteristics to positions on the path. Poor and minority children are slow and immature. Another rhetorical move in this paragraph is to conflate poor with minority, not only by using them in connection with each other, but also by occasionally using one to stand for the other. The authors wrote that:

> Overall, the research shows that *poor and minority* children entering school do possess some informal mathematical abilities, but that many of these abilities have developed at *slower rates* than *middle-class* children.

In the second half of this sentence, the authors qualify the noun 'children' with only the adjective 'middle-class', although they began the sentence by referring to 'poor and minority children'. This is a little sleight of hand (or text). Consider the alternative: poor and minority children develop at slower rates than white children.

Replacing 'white' with 'middle-class' makes the sentence less jarring, but by using 'poor and minority' at the beginning of the sentence and by repeating the same two adjectives in the following sentence, the authors order race along with income-level and mathematical ability. In addition, words like 'poor', 'minority', 'immature', and 'slow' become attached to each other in ways that any one can stand as a proxy for the others. When multiple texts attach (however circuitously) concepts that typically would be considered categorical rather than ordered—such as race and gender—onto metaphors of hierarchy, they work to create discursive expectations. The result is that when the authors of the US NCTM *Principles and Standards for School Mathematics* (2000) wrote that 'all students, regardless of their personal characteristics, backgrounds, or physical challenges, must have opportunities to study—and support to learn—mathematics' (p. 12), readers can understand this to mean that some students—those who 'live in poverty, students who are not native speakers of English, students with disabilities, females, and many non-white students' (p. 13) can be thought of as struggling, or perhaps behind. Through similar reiterations, the metaphor of students distributed along a path begins to describe not just individual children, but kinds of children. I contend that as a result of these layerings of discourse, Sara, as well as most other educators, would have been inclined to position the students from Blythe as 'behind' the students from Northside because this ordering had more coherence with other metaphors of hierarchy within the discourse of mathematics education than the reverse order. That is, it would have been conceptually surprising to think of a class of mostly minority students from poorer homes as 'ahead of' a class of mostly majority students from middle-class homes.

Possibilities for teacher educators

This is not an argument intended to absolve Sara, or any educator, of responsibility for their words. In fact, it is just the opposite. Seeing Sara's positioning of her students as the result of dense metaphors in mathematics education rather than as the problematic beliefs of an individual changes the responsibility of teacher educators. Instead of (or perhaps, in addition to) searching for ways to change beginning teachers, we as teacher educators may think about the ways that our own teaching, writing, and conversation work for or against hierarchical ways of thinking about children. In addition, we can be conscious of the ways that we use demographic labels. This task is not easy. As Lakoff and Johnson (1980: 4) wrote, metaphors are not 'just *talk*'; they influence how people understand the world, and with dense metaphors, like learning mathematics as travel along a narrow path, re-envisioning the world requires new words, new metaphors, and new practices that may seem troubling, precisely because they are at odds with the ways that we as educators have always understood mathematics and children. Below I suggest some possibilities for reconceptualizing our work as educators and as researchers in order to create new ways of thinking about learning, children, and mathematics.

First, we may imagine new metaphors for talking about the work that we want schools to do. Learning, and not just in mathematics, may seem necessarily to contain within it the notion of progress and of more or less, which leads to the ordering of mathematical concepts and the ranking of children. A long history of (and a current emphasis on) achievement testing as *the* measure of learning contributes to the metaphor of learning as travel on a path. One way of disrupting the path metaphor may be to describe learning as diversification rather than progress. For example, Gardner's (1983, 1999) work on multiple intelligences offered one possible language for talking about intelligence without ordering. By identifying specific ways of thinking, such as visual, logical, and linguistic, Gardner argued that intelligence is not one continuum, but a diverse 'grab-bag' of abilities. Drawing on this metaphor, learning may be looked at as a practice that adds diversity to one's grab-bag. In mathematics, where visualizing, making sense of the world quantitatively, arguing, asking questions, creating representations, following procedures, and much more, can all be seen as essential, drawing on metaphors that value diverse ways of working with number, data, and space ought to be possible. The metaphor of a grab-bag, or perhaps a toolbox, could challenge current thinking about standards and assessments. For example, the toolbox metaphor calls into question the idea of a composite score. Each student can be seen as having certain tools that he or she can use effectively; each can be seen as requiring opportunities to learn to use new tools or to practise with unfamiliar ones. However, a student's proficiency with a hammer ought not be seen as a proxy for his or her ability to use a wrench. Similarly, it is conceptually difficult to rank tools. What a student should learn next depends on his or her interests and the job at hand. Unlike the metaphor of a path, which carries with it the idea of a logical and necessary series of steps that each person must follow, the metaphor of the toolbox opens possibilities for multiple ways of interacting with a discipline. No doubt there are other metaphors that may work in similar ways.

Second, we as educators in the US can become more conscious of the ways that we invoke categorical language related to race, ethnicity, and social class. Schooling resources, including money, challenging curricula, and skilled teachers, have historically not been equally distributed across all demographic groups of children. Acts of racism, sexism, and discrimination against the poor have occurred and will occur again. School cultures can vary in the ways they welcome children, depending on their lives at home, and as researchers we need a language to talk about these phenomena. However, by repeatedly singling out certain demographic groups when talking about difficulties in learning mathematics, we create a shared understanding that some groups, but not others, have trouble learning. One rhetorical strategy may be consciously to create 'incoherence' in our writing, for example by writing 'boys as well as girls', 'minority students as well as majority students'. Such small changes may at least challenge some of the unconscious connections drawn in a lot of writing. Another strategy may be to become more precise in our research so that we write not about low-income, minority students, but about particular students in particular places—who have certain language practices, schooling histories, family lives, friends, and personal preferences, in addition to races and socioeconomic statuses. The more complicated students become in our discourse the more difficult they will become to rank based on any one characteristic.

Finally, educational researchers need to continue to consider the ways that the commonly used theories and methods add density to metaphors of hierarchy. When we choose to emphasize the individual through studies that draw on psychological notions of development or that create expectations for individuals based on some theoretical category, such as 'grade 3 students', '10-year-olds', or 'girls', we create more opportunities to rank and order children. When theories, like the van Hiele levels, are repeated in classrooms and textbooks, they take on the power of physical law. Students who do not achieve stages on time or in the correct order are seen as requiring remediation. Rather than labelling poor and minority children as 'immature' because they, as a group, do not solve problems in similar ways to middle-class, majority children, we may instead explore these differences as a way to understand the variety of ways that children come to know the world quantitatively. We may, in fact, expect that it would be impossible to articulate or to study what a 'grade 3 student' could (or could not) do. For some time, literacy researchers[7] have moved away from talking about differences in the ways that children come to the written and spoken language as evidence of maturity (or not). Instead, they have documented differences with the goal of showing the many ways that children move toward understanding. As Dyson (2003: 5) wrote:

> I hope to turn this [developmental] view inside out, as I look from inside a particular child culture out toward school demands. In this way, I aim to provide conceptual substance for a different theoretical view of written language development, one that normalizes variations in (as well as broadens conceptions of) children's literacy resources and learning pathways.

Similar work in mathematics may do more to work toward equity-oriented outcomes than further efforts to remediate the immature.

Moving away from psychology-inspired questions, theories, and methods that seek to describe and order individuals may reduce the density of metaphors of hierarchy in the educational discourse. Studies that seek to explore the 'social soup' in which all human interactions are located work against ordering because the unit of analysis moves beyond the individual. Rather than comparing individuals on the basis of their performance, beliefs, or demographics, these studies may explore the current theories, physical worlds, and practices in which we are all located. In particular, rhetorical studies, which draw on literary and philosophical traditions, rather than on social science, may disrupt the creation of new hierarchies. Social science, with its data, codes, and findings, often creates new ways of categorizing children, teaching, and schools. In addition, because this process is 'scientific', these categories (which are often implicitly or explicitly ranked) seem quite real. Rhetorical traditions, because they do not call on science to add persuasive power to their arguments, are less likely to produce firm categories backed up by data. For example, in this paper, I did not classify all metaphors used in mathematics education, link kinds of people to the metaphors they use, or 'prove' that metaphors of hierarchy are harmful to students. The argument is more permeable than that, more open to challenges, less narrowing to other possibilities. By drawing on literary rather than scientific traditions, rhetoric offers researchers the opportunity to talk about the world in ways that cause others to think differently about social interactions without etching boundaries between people as the result of 'evidence'. Increasing uncertainty and permeability in educational research necessarily works against hierarchical thinking.

I acknowledge my uncertainty in making these suggestions. Any of the actions suggested above could operate in precisely the opposite way to that I anticipate. Like everyone else, I am immersed in the discourse I am trying to change. The primary intervention of this paper is to make visible metaphors of hierarchy that previously may have gone unnoticed. As Santa Ana (1999: 217) pointed out, the more common our metaphors are, the less likely we are to question their usefulness and accuracy. As long as our language of ranking remains invisible, it works to reinscribe a particular set of power relations—one in which some students (often identifiable by physical characteristics) are seen as mathematically capable and others are not. By naming these metaphors, as well as their discursive consequences, it becomes possible to think about—and to work toward—other possibilities.

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Notes

1. Names of students, teachers, and schools are pseudonyms.
2. My goal here is not to say that Sara did not see the lesson for what it 'really' was, but to point out that she constructed differences that were meaningful to her. I also saw differences between this lesson and those in Diana's classroom: the teacher talked far more, students did no independent work, and children almost never spoke to each other. My production of these differences could equally serve as a point of departure for study. That, however, remains a challenge for another day.
3. See, e.g. Barlow and Cates (2006), Mingus and Grassl (1999), Stipek *et al.* (2001), Timmerman (2004), Warfield *et al.* (2005), and Wilkins and Brand (2004).
4. See, e.g. Artiles and McClafferty (1998), Bondy *et al.* (1993), Brown (2004), and Sleeter (2001).
5. See, e.g. Lave (1988), Rogoff (1995), and Wenger (1998).
6. When I think of metaphors, our lives as 'social soup' is one of my favourites. Rather than seeing individuals and discourse/society as separate, the soup metaphor merges these two constructs. Although it is possible to call individuals or discourse to the foreground (as a chef might consider the amount of salt or potatoes), the soup cannot be separated from its component parts. Nor can we, as ingredients, get a vantage point outside of the soup to make observations.
7. See, e.g. Cazden (1988), Dyson (1995, 1997, 2003), and Heath (1983).

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